Foundation of the lattice Boltzmann method

1 Lattice Boltzmann method is …

- Lattice Boltzmann method(LBM) simulates the fluid motion process by
  - assuming a collection of particles to be distribution functions
  - relying on collision-and-stream behavior of numerous molecules
- LBM shows more promise in high-speed LES simulations for complex and large-scale urban flows
  - significantly simpler algorithm
  - appropriation of parallel computation nature

2 Theory of LBM

- LBM is based on the mesoscopic lattice Boltzmann equation (LBE)

$$f_a(\mathbf{r}+\delta\mathbf{e}_a, t+\delta) - f_a(\mathbf{r},t) = \Omega[f_a(\mathbf{r},t)]$$

- Relaxation time schemes
  - Single relaxation time scheme (SRT)

$$\Omega[f_a(\mathbf{r},t)] = -\frac{1}{\tau}\left[f_a(\mathbf{r},t) - f_a^{eq}(\mathbf{r},t)\right]$$

  - Multi relaxation time scheme (MRT)

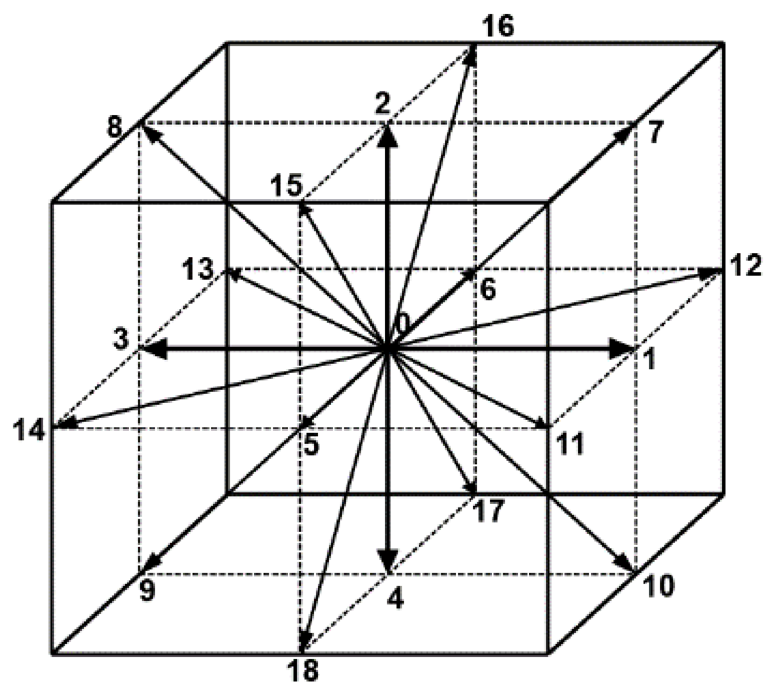
$$\Omega[f_a(\mathbf{r},t)] = -\mathbf{M}^{-1}\mathbf{S}\mathbf{M}[f_a(\mathbf{r},t) - f_a^{\mathrm{eq}}(\mathbf{r},t)]$$

f_a : distribution function for particle a
f_a^{eq} : equilibrium distribution function for particle a
$\mathbf{r}$: spatial position of the particle
$\mathbf{e}_a$: discrete velocity for particle a
t : time
δ : discrete time step
Ω : collision function

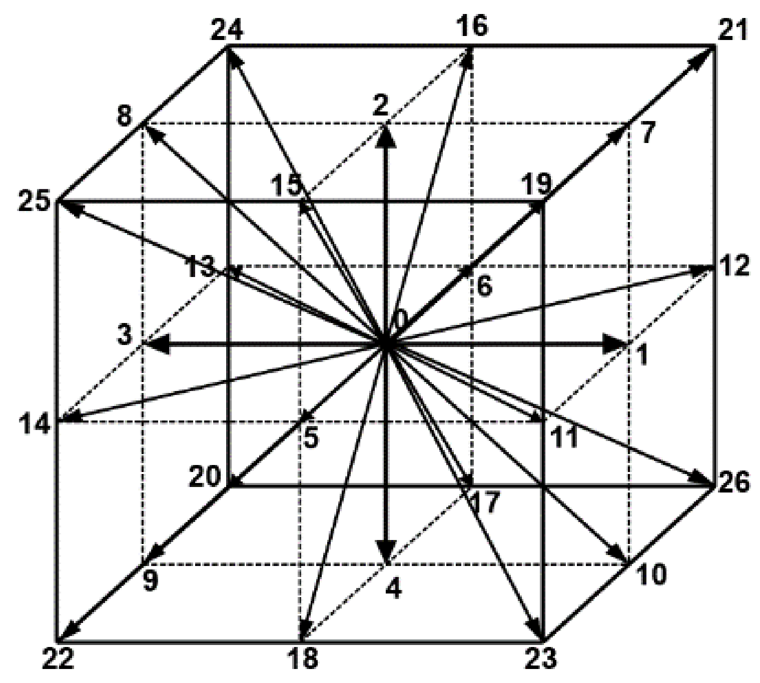
τ : relaxation time
$\mathbf{M}$: matrix structured to transform distribution functions to moments
$\mathbf{M}^{-1}$: Inverse matrix of $\mathbf{M}$
$\mathbf{S}$: relaxation coefficient diagonal matrix corresponding to $\mathbf{M}$

3 Discrete velocity scheme

- D319 and D3Q27 discrete speed schemes are employed in the built environment



D3Q19 scheme



D3Q27 scheme

4 Large-eddy simulation in LBM

- LBM can be used with a large-eddy simulation (LBM-LES) model in built environment
- Viscosity is constituted of the molecular viscosity ν and the eddy viscosity ν_t by Smagorinsky SGS model
  - $\nu_t = (c_k\Delta)^2|\bar{S}|$ (eddy viscosity in the LES theory)
  - $\nu_{total} = {e_s}^2(\tau_{total} - 0.5)\delta$ (total viscosity in the LBM theory)

e_s : acoustics speed on the lattice
δ : discrete time step
Δ : filter width

τ : relaxation time
c_k : Smagorinsky constant
$\bar{s}$: strain tensor

- τ_{total} substitutes the relaxation time τ in the LBE to implement the LES simulation

Benchmark of LBM for the indoor isolated flow

1 Objective

- Implement simulations of 2D indoor isothermal flow using LBM and FVM with LES
  - benchmark LBM's accuracy
  - compare the accuracy of the LBM and FVM
- Exam LBM's applicability in indoor airflow

2 Boundary conditions

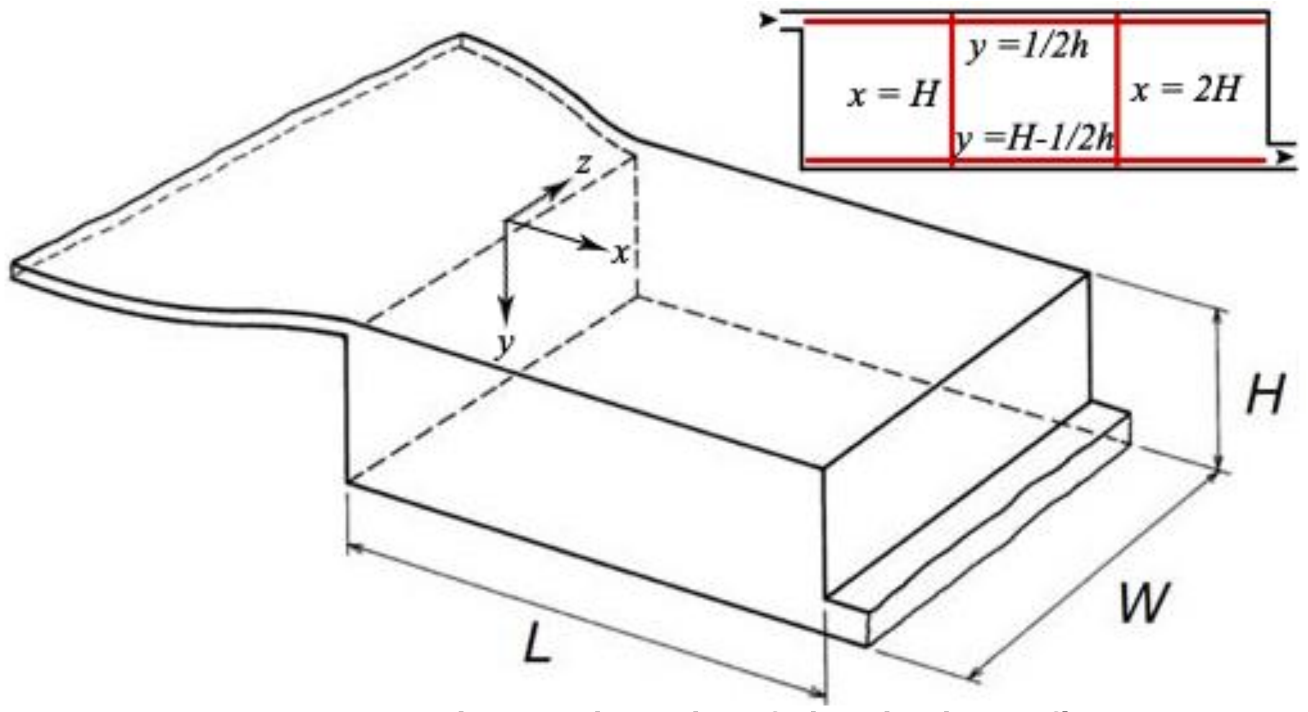


Fig.1 Sketch of the indoor flow case

Table 1 Simulation conditions

Item	FVM-LES	LBM-LES
Software	OpenFOAM	OpenLB
Sub-grid scale model	standard Smagorinsky model (C_s = 0.12)	
Time discretization	Euler-implicit	-
Space discretization	2^{nd}-order central difference	-
Lid B.C.	Uniform velocity boundary,	
Outlet B.C.	Velocity Gradient-zero, t=0.16 H	
Other B.C.	wall function (Spalding's law)	Bounce-back condition

3 Results of time-averaged scalar velocity

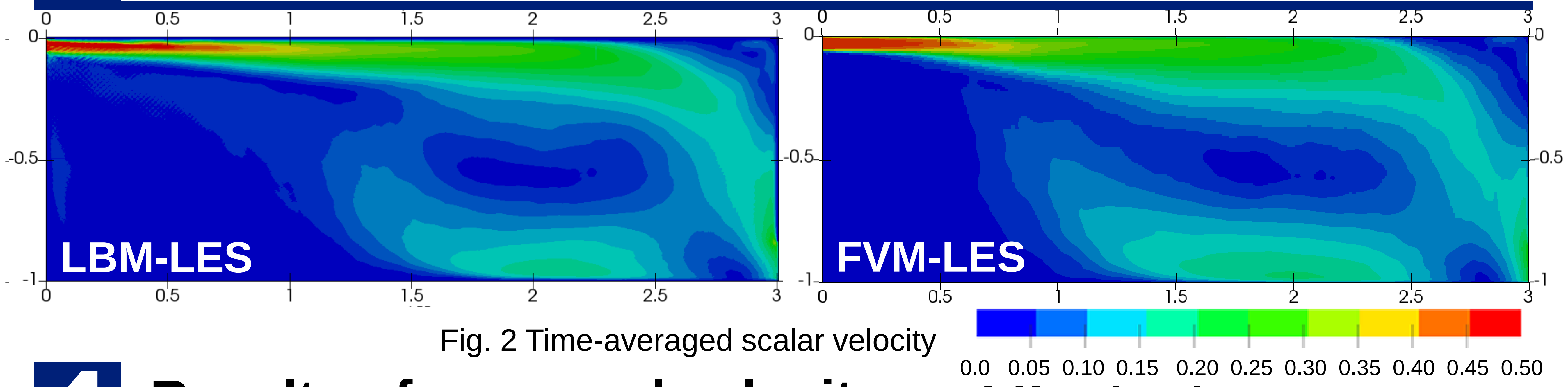


Fig. 2 Time-averaged scalar velocity

4 Results of averaged velocity and fluctuations

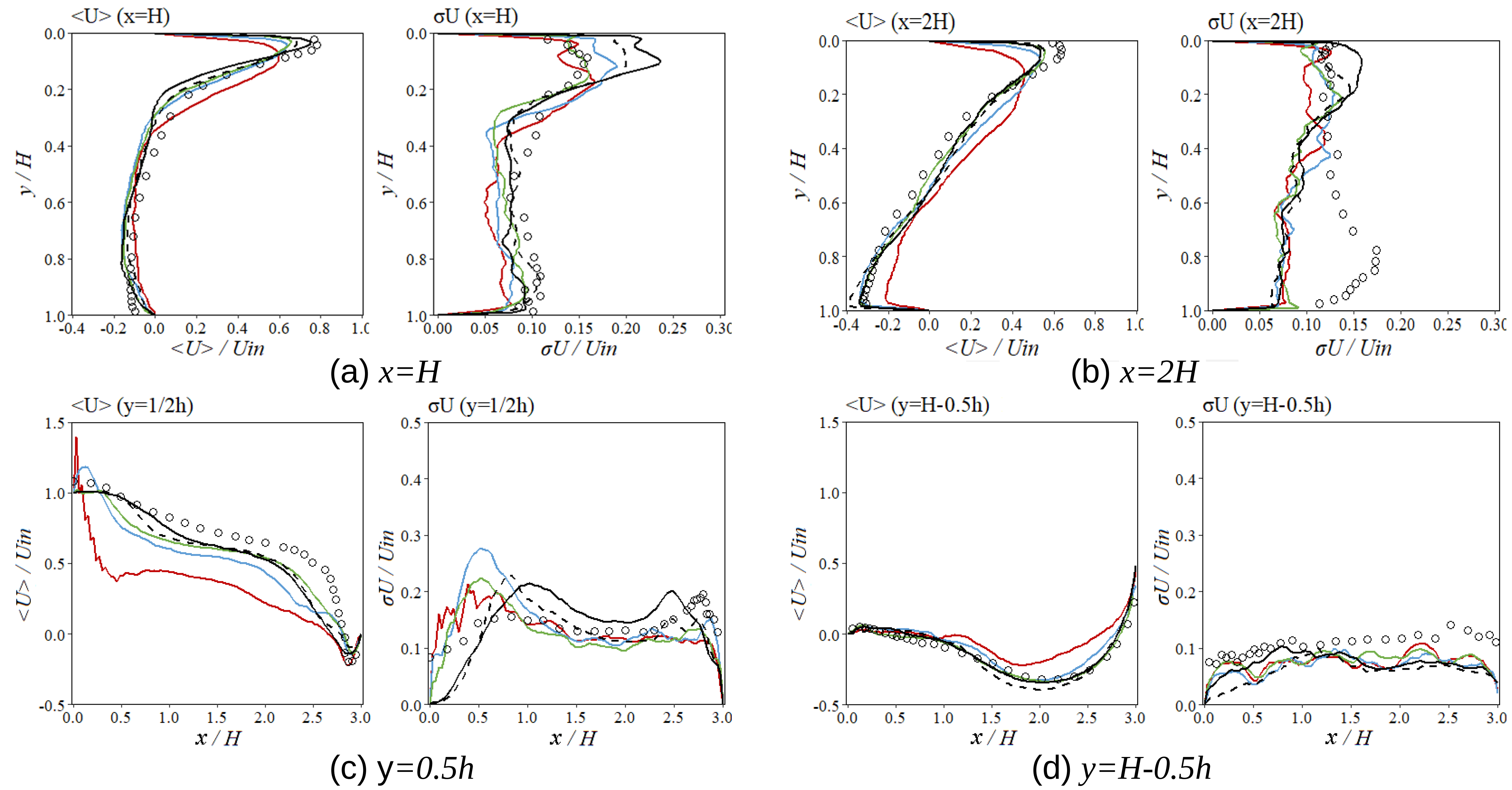


Fig. 3 Distribution of streamwise component of time-averaged velocity (left) and the standard deviation of fluctuating velocity (right)

- LBM simulates the physical flow and the experimental data in indoor flow like FVM.

大岡研究室・菊本研究室
Ooka Lab., and Kikumoto Lab.

Benchmark of LBM for the outdoor isolated flow (part 1)

1 Objective

- ➢ Implement simulations of flow around a high-rise building using LBM and FVM with LES to exam LBM's applicability in outdoor airflow
  - verify the accuracy of the LBM and FVM
  - compare different relaxation time and discrete velocity schemes
- ➢ Compare computation speeds and parallel computation performances

2 Simulation target and case settings

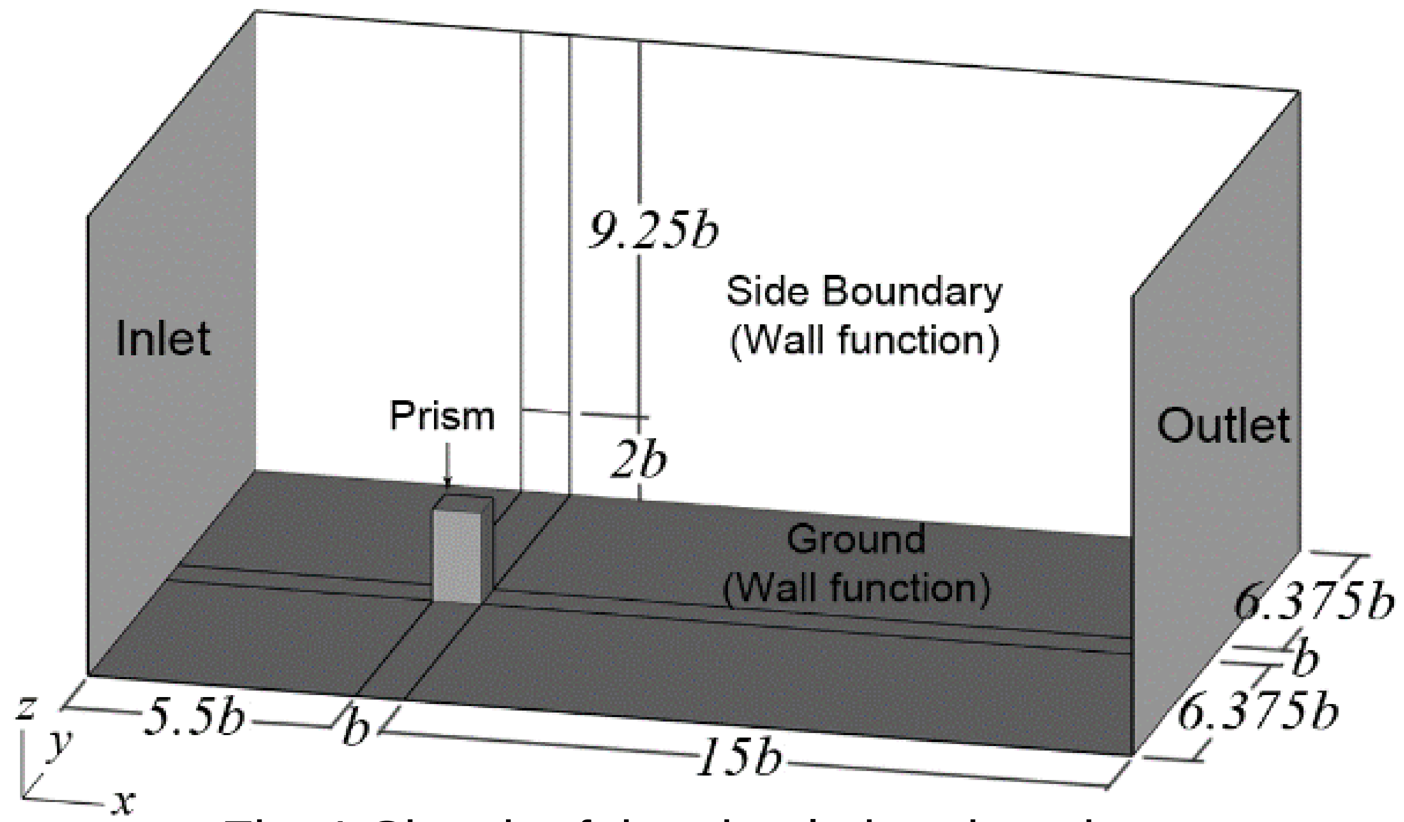


Fig. 1 Sketch of the simulation domain

Table 1 Case Settings

Case Name	Calculation Method	Relaxation time scheme	Discrete velocity scheme	Mesh size
LBM_02_SRT_D3Q19	LBM-LES	SRT(BGK)	D3Q19	0.02m (1/8 b)
LBM_01_SRT_D3Q19		SRT(BGK)	D3Q19	0.01m (1/16 b)
LBM_01_SRT_ D3Q27		SRT(BGK)	D3Q27	0.01m (1/16 b)
LBM_01_MRT_ D3Q19		MRT	D3Q19	0.01m (1/16 b)
FVM_02	FVM-LES	-	-	0.02m (1/8 b)

3 Results of time-averaged scalar velocity

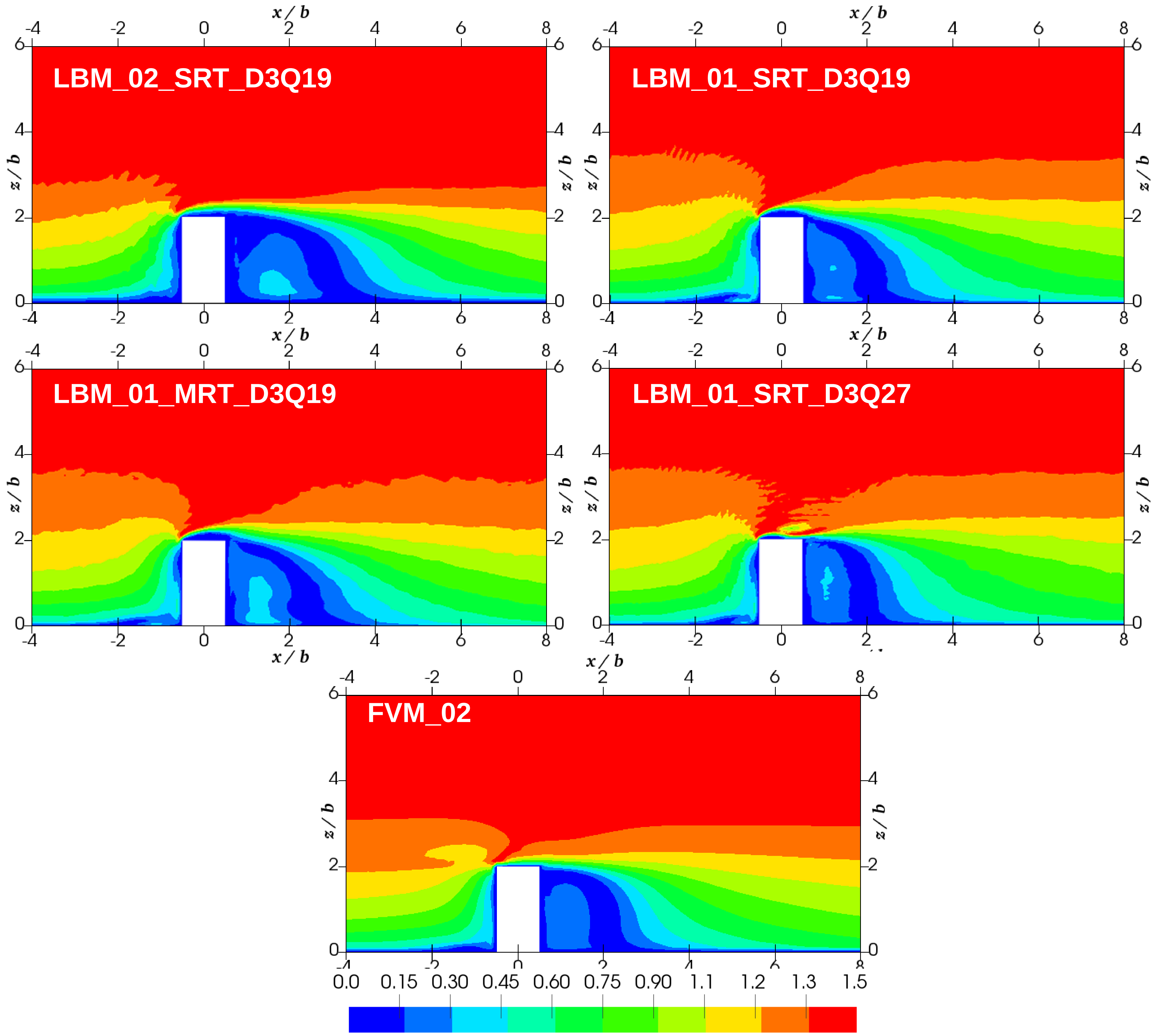


Fig. 2 Time-averaged scalar velocity of all cases

- ➢ LBM adequately simulates the physical flow features in outdoor flow like FVM.

Benchmark of LBM for the outdoor isolated flow (part 2)

4 Comparisons for velocity and turbulent kinetic energy

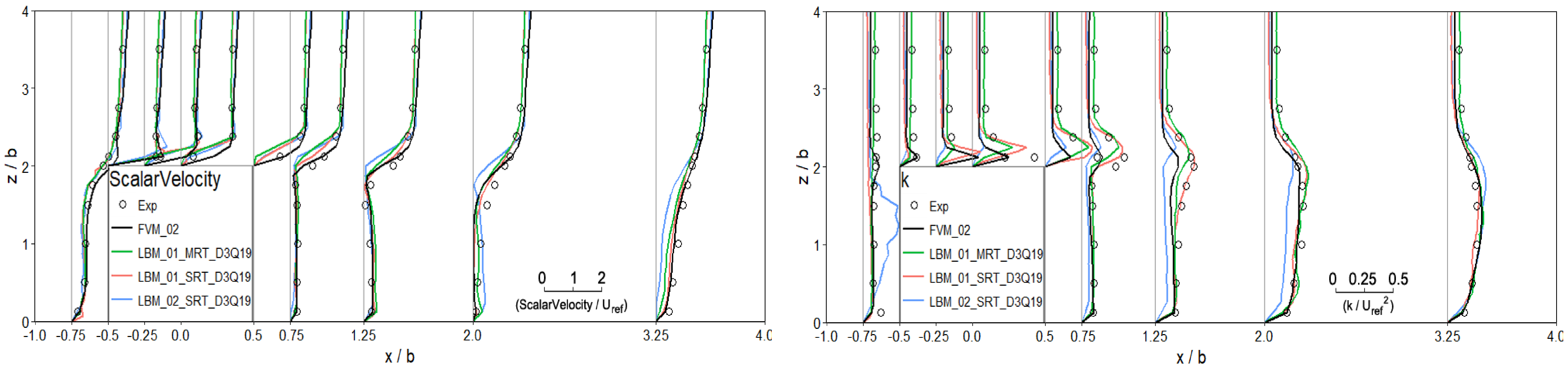


Fig. 3 Time-averaged scalar velocity (left) and turbulent kinetic energy (right) on the vertical section

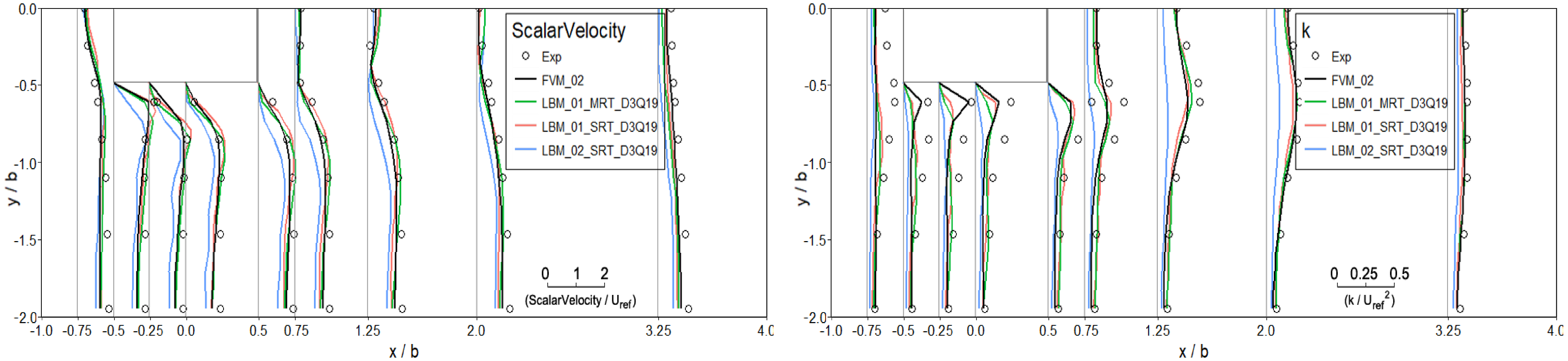
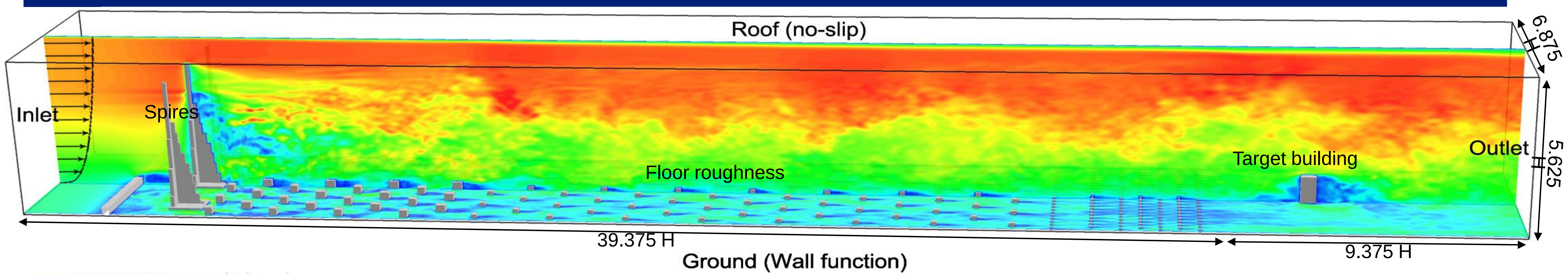


Fig. 4 Time-averaged scalar velocity (left) and turbulent kinetic energy (right) on the horizontal plane at z/b =1/8

6 Results of instantaneous velocity



5 Computational speed

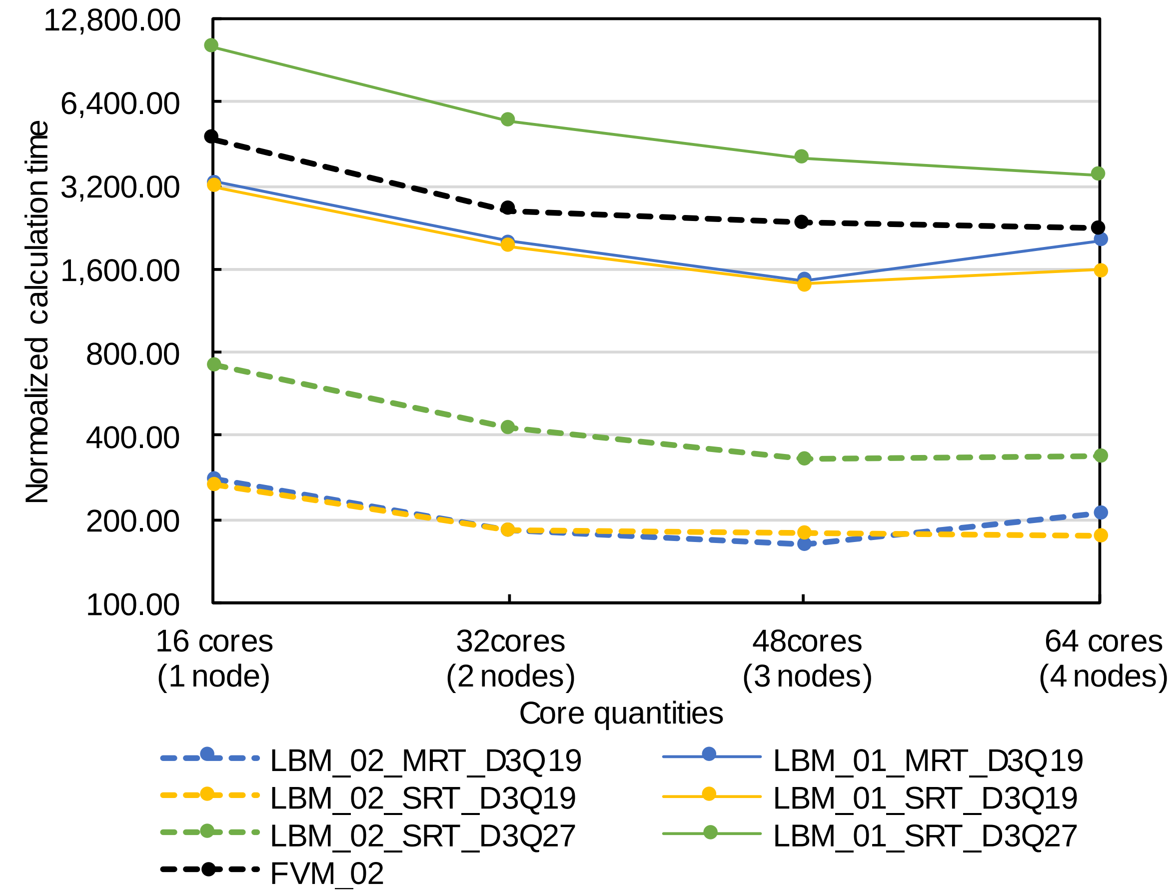


Fig. 5 Normalized computation time of all cases

- LBM's speed is larger than that of FVM under the same result accuracy

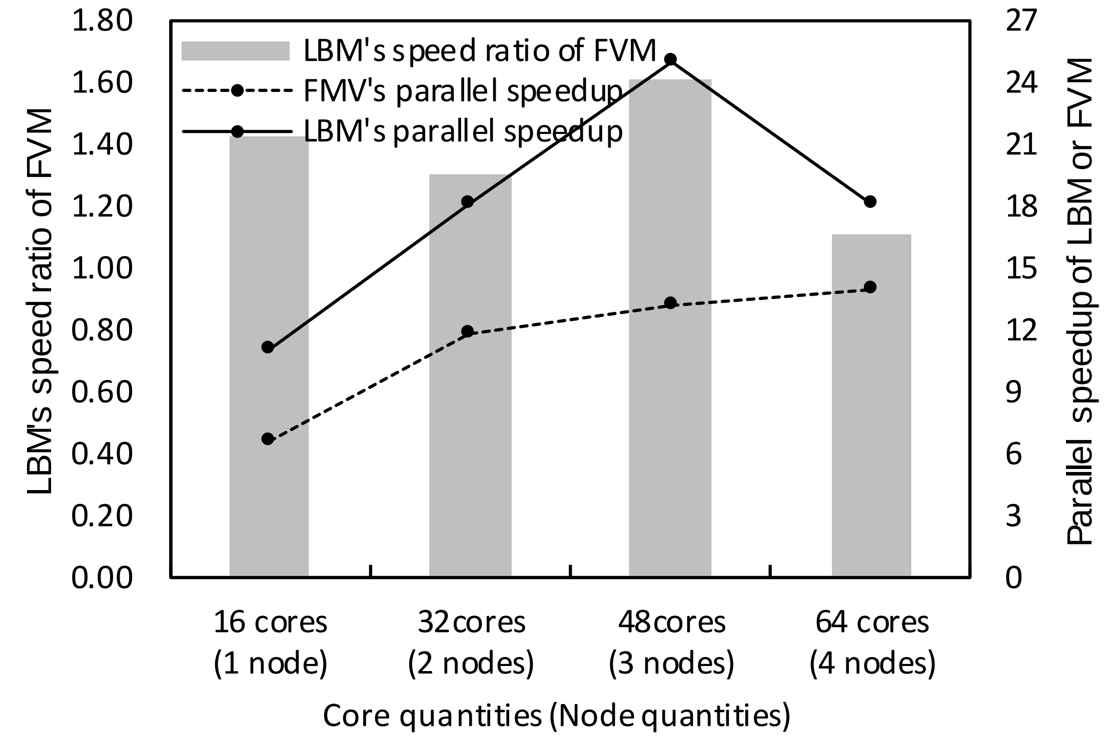


Fig. 6 Parallel computational performance

- LBM's parallel computational performance is more significant than FVM

大岡研究室・菊本研究室
Ooka Lab., and Kikumoto Lab.